Steady film condensation and boiling adjacent to a body of arbitrary shape in a porous medium

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The problems of steady film condensation and boiling over a body of arbitrary shape embedded in a porous medium have been attacked by means of the similarity transformation. A new similarity variable is suggested to solve the problems once for all possible two-dimensional and axisymmetric bodies of arbitrary geometrical configuration. Upon transforming the governing equations and boundary conditions using the similarity variables, the resulting set of equations transforms into that obtained for the case of a vertical flat plate. Thus, the numerical values furnished by Parmentier and Cheng for a flat surface are readily applicable to two-phase flows over two-dimensional and axisymmetric bodies of arbitrary shape.

Keywords: *film condensation, film boiling, porous medium, arbitrary shape, similarity solution*

Introduction

The study of two-phase flow in a porous medium involving phase change has a number of important applications in both engineering and the earth sciences. Parmentier[1] analysed film boiling on a heated vertical surface in a porous medium to investigate the movement of ground water in permeable rock surrounding igneous intrusions near the surface of the earth. The problem of film condensation, which has much in common with that of film boiling, was treated by Cheng [2] in view of its applications in geothermal energy utilization. These studies, however, are restricted to the case of a simple geometry, namely: a flat surface (or a cone).

The present paper proposes a general similarity transformation procedure appropriate for the problems of steady film boiling and condensation over a two-dimensional or axisymmetric body of arbitrary shape embedded in a porous medium. A transformed variable similar to the one proposed by Merkin[3] for single-phase free convection in a porous medium, and subsequently extended by the authors[4] to the case of free convection over a nonisothermal body of arbitrary shape, is introduced to account for possible geometric effects on the boundary layer length scale. It is shown that, by virtue of this transformation, the governing equations and boundary conditions for bodies of arbitrary shape can be reduced to those for a vertical flat plate, which have already been solved by Parmentier[1] and Cheng[2].

Analysis

In the previous studies[1,2], certain simplifying assumptions were introduced to make the problems tractable within the scope of the classical boundary layer concept. The same assumptions will also be adopted in the present analysis.

Upon projecting possible phase change paths associated with steady film boiling into the pressure–temperature space, Parmentier justified the assumption that a phase boundary occurs across which liquid (vapour) is transformed directly to vapour (liquid) without the formation of a mixed phase region. Thus, the liquid and vapour are separated by a distinct boundary with no intermediate two-phase zone. It is also assumed that the properties within a film adjacent to the body are constant, while the fluid outside the film is at its saturation temperature. Moreover, the film is assumed to be sufficiently thin so that the boundary layer approximations may be exploited.

Since the single-phase boundary layer equations can be applied separately to the vapour and the liquid, the problems of film boiling and condensation need be treated only once for both (as we interchange the roles of the liquid and the vapour). Thus, we direct our attention to the problem of film condensation over two-dimensional and axisymmetric bodies of arbitrary geometrical configuration. (For the case of subcooled film boiling, the compressibility of the surrounding subcooled liquid must be taken into consideration as in the analysis of Cheng and Verma[5], since the effect of free convection on heat transfer rate may no longer be negligible.)

The physical model and boundary layer coordinates (x, y) are indicated in Fig 1. The body under consideration may be either plane or axisymmetric, and its geometry is specified by the function $r(x)$. The cooled body, with surface temperature T_w, is embedded in a porous medium filled with a dry saturated vapour at a saturation temperature T_s (corresponding to its pressure). Thus, both a liquid and vapour layers develop simultaneously as steady condensation takes place over the isothermal wall.

The set of the governing equations—namely, the continuity equation, the Darcy's law and the energy equation—may be given for each phase as follows.

For the liquid (condensate):

$$\frac{\partial}{\partial x} r^* u + r^* \frac{\partial}{\partial y} v = 0 \qquad (1)$$

$$u = \frac{K}{\nu} g_x \qquad (2)$$

and

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \qquad (3)$$

For the vapour:

$$\frac{\partial}{\partial y} v_G = 0 \qquad (4)$$

$$u_G = 0 \qquad (5)$$

$$T_G = T_s \qquad (6)$$

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where

$$r^*=\begin{cases}1 & \text{for plane flow}\\ r(x) & \text{for axisymmetric flow}\end{cases} \tag{7}$$

and

$$g_x=g\left[1-\left(\frac{\mathrm{d}r}{\mathrm{d}x}\right)^2\right]^{1/2}\frac{\rho-\rho_G}{\rho} \tag{8}$$

The boundary and matching conditions are as follows. At the wall surface ($y=0$):

$$v=0, \qquad T=T_w \tag{9a,b}$$

At the vapour–liquid interface ($y=\delta$):

$$T=T_s \tag{10a}$$

$$\rho u\frac{\mathrm{d}\delta}{\mathrm{d}x}-\rho v=-\rho_G v_G \tag{10b}$$

$$\left(\rho u\frac{\mathrm{d}\delta}{\mathrm{d}x}-\rho v\right)h_{LG}=k\frac{\partial T}{\partial y} \tag{10c}$$

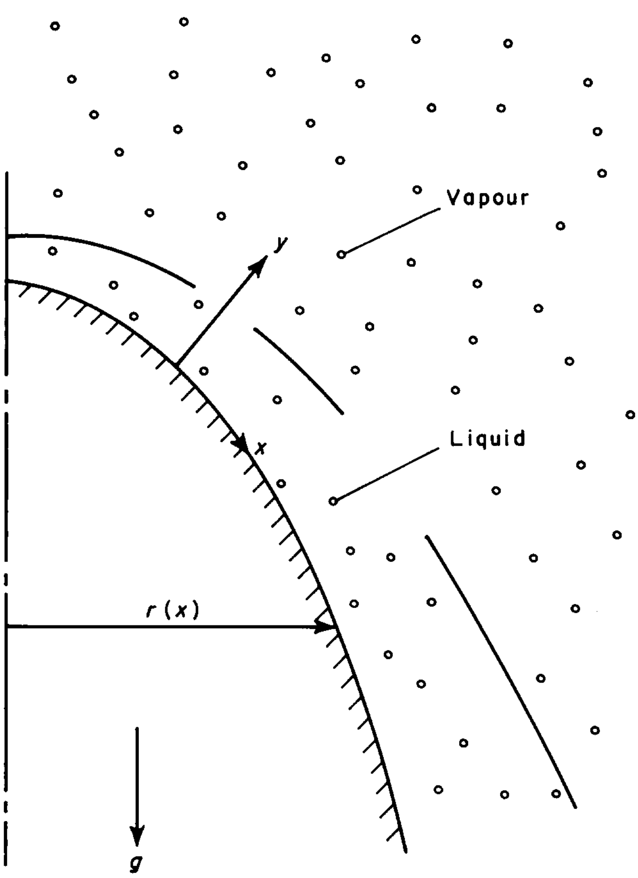

Figure 1 Physical model and coordinates

In the preceding equations, u and v are the Darcian velocity components in the x and y directions while T is the local temperature. The tangential component of the gravitational acceleration g is indicated by g_x, which is related to the local surface inclination through Eq (8). Furthermore, K is the permeability, ν the kinematic viscosity, α the equivalent thermal diffusivity, and k the equivalent thermal conductivity. The subscript G denotes the quantities associated with the ambient vapour phase, while no subscript is assigned for the liquid phase. h_{LG} is the latent heat of vapourization.

Eqs (5) and (6) for the saturated vapour obviously satisfy Darcy's law and the energy conservation principle, respectively. Therefore, the differential equations (1), (3) and (4) must be solved with the Darcy's equation (2) for the condensate and the conditions given by Eqs (9a) to (10c). Note that the problem is not overspecified, since the film thickness δ is unknown.

Let us introduce the stream function ψ such that the continuity equation (1) may automatically be satisfied:

$$u=\frac{1}{r^*}\frac{\partial\psi}{\partial y} \tag{11a}$$

$$v=-\frac{1}{r^*}\frac{\partial\psi}{\partial x} \tag{11b}$$

Merkin[3] introduced a similarity transformation for the single-phase problem of free convection in a porous medium. His transformation has been generalized by the authors[4] for the case of a nonisothermal body of arbitrary shape. These transformations may be extended to the present two-phase flow problem as

$$\psi=\alpha r^*(Ra_x I)^{1/2}f(\eta) \tag{12a}$$

$$T-T_s=\theta(\eta)(T_w-T_s) \tag{12b}$$

$$\eta=\frac{y}{x}(Ra_x/I)^{1/2} \tag{12c}$$

where

$$Ra_x=\frac{Kg_x x}{\nu\alpha} \tag{13a}$$

$$I(x)=\frac{\int_0^x g_x r^{*2}\,\mathrm{d}x}{g_x r^{*2}x} \tag{13b}$$

Ra_x is the local Rayleigh number, while η is the proposed similarity variable. The function I as defined by Eq (13b) adjusts the scale in the normal direction according to a given body geometry.

In terms of these transformed variables, the Darcian velocities in the condensate layer are

$$u=\frac{\alpha}{x}Ra_x f' \tag{14a}$$

and

$$v=-\frac{\alpha}{x}(Ra_x/I)^{1/2}\left[\left(I\frac{\mathrm{d}\ln g_x r^*}{\mathrm{d}\ln x}-\frac{1}{2}\right)\eta f'+\frac{1}{2}f\right] \tag{14b}$$

Notation

f	Dimensionless stream function
g	Gravitational acceleration
h_{LG}	Latent heat of vapourization
I	Function associated with a body geometry
k	Thermal conductivity
K	Permeability
Nu_x	Local Nusselt number
q_w	Local surface heat flux
r	Function representing wall geometry
r^*	1 for plane flow and r for axisymmetric flow
Ra_x	Local Rayleigh number
T	Temperature
u, v	Darcian velocity components
x, y	Boundary layer coordinates
α	Thermal diffusivity
γ	Ratio of a horizontal axis to a vertical axis
η	Similarity variable
θ	Dimensionless temperature
ν	Kinematic viscosity
ρ	Density
ψ	Stream function
Subscripts	
G	Vapour
w	Wall
r	Reference

where the primes are the differentiation with respect to η. Substitution of Eq (14a) into Eq (2) yields

$$f'=1 \tag{15a}$$

This may readily be integrated as

$$f=\eta \tag{15b}$$

using the boundary condition

$$f(0)=0 \tag{15c}$$

which is equivalent to Eq (9a) with (14b) substituted.

The preceding results may be substituted into the energy equation (3). After a considerable manipulation, we obtain a remarkably simple expression as

$$\theta''+\tfrac{1}{2}\eta\theta'=0 \tag{16}$$

The foregoing equation is subject to the boundary conditions, namely:

$$\theta(0)=1 \tag{17a}$$

$$\theta(\eta_\delta)=0 \tag{17b}$$

which are equivalent to Eqs (9b) and (10a), respectively. The preceding equation (16) with Eqs (17) has the exact solution given by

$$\theta(\eta)=1-\frac{\operatorname{erf}(\eta/2)}{\operatorname{erf}(\eta_\delta/2)} \tag{18a}$$

where

$$\eta_\delta=\frac{\delta}{x}(Ra_x/I)^{1/2} \tag{18b}$$

The unknown η_δ (or δ) should be determined from the matching condition given by Eq (10c) which, after a considerable manipulation, reduces to

$$Ja=\frac{k(T_s-T_W)}{\rho\alpha h_{LG}}=-\eta_\delta/2\theta'(\eta_\delta)$$
$$=\frac{\pi^{1/2}}{2}\eta_\delta\exp(\eta_\delta^2/4)\operatorname{erf}(\eta_\delta/2) \tag{19}$$

where Ja is the Jacob number associated with the degree of subcooling.

Once the temperature profile (given by Eq (18a)) is known in this way, the local Nusselt number of interest can be evaluated from

$$Nu_x=\frac{q_w x}{k(T_s-T_w)}=-\theta'(0)(Ra_x/I)^{1/2}$$
$$=\frac{(Ra_x/I)^{1/2}}{\pi^{1/2}\operatorname{erf}(\eta_\delta/2)} \tag{20}$$

where q_w is the local surface heat flux.

Finally, let us consider the vapour phase. The continuity equation (4) may readily be integrated using the last matching condition given by Eq (10b). Hence, the velocity of the vapour moving towards the interface is given by

$$v_G=-\frac{\rho}{2\rho_G}\frac{\alpha}{x}(Ra_x/I)^{1/2}\eta_\delta \tag{21}$$

It is most interesting to note that the resulting equations (18a) and (19) are identical to those obtained by Cheng[2] for a flat surface. Thus, the numerical values of η_δ as a function of Ja furnished by Cheng (solving Eq (19)) are directly available for all possible two-dimensional and axisymmetric bodies.

Results and discussion

The characteristic equation (19) indicates that an increase in Ja (degree of subcooling) results in thickening of the film. Of special interest is the limiting case of $\eta_\delta \to 0$, where

$$\eta_\delta=(2Ja)^{1/2}$$

The results of numerical computations are presented in Cheng's paper[2] in terms of Nu_x and Ja. Cheng also established an approximate expression for Nu_x by examining his numerical results. This useful approximate expression may be generalized to the present case of arbitrary geometrical configuration as

$$Nu_x=\left(\frac{1}{2Ja}+\frac{1}{\pi}\right)^{1/2}(Ra_x/I)^{1/2} \tag{22}$$

which is accurate to the fourth significant figure when compared with the exact solution.

By virtue of the proposed transformation, the results on a vertical flat plate can be translated to any particular two-dimensional or axisymmetric body of arbitrary shape. This can be done by evaluating the function I for the given geometry. For example, in the cases of flat plates, vertical cones, horizontal ellipses and ellipsoids (including a horizontal circular cylinder and a sphere), we evaluate I according to its definition given by Eq (13b) as

$$\frac{1}{I}=\begin{cases}1 & \text{for vertical plates}\\ 3 & \text{for vertical cones pointing upward}\\ \dfrac{(x/L_r)\sin\phi}{(1-\cos\phi)(\sin^2\phi+\gamma^2\cos^2\phi)^{1/2}} & \text{for horizontal ellipses}\\ \dfrac{3(x/L_r)\sin^3\phi}{(\cos^3\phi-3\cos\phi+2)(\sin^2\phi+\gamma^2\cos^2\phi)^{1/2}} & \text{for ellipsoids}\end{cases} \tag{23}$$

where

$$\phi=\sin^{-1}(r(x)/\gamma L_r) \tag{24a}$$

and

$$x/L_r=\int_0^\phi(\sin^2\phi+\gamma^2\cos^2\phi)^{1/2}\,d\phi \tag{24b}$$

The upper (front) and lower (rear) stagnation points are located at $\phi=0$ and π, respectively. The symbol L_r denotes reference lengths such as a plate height and a vertical semi-axis of an ellipse or an ellipsoid, while γ stands for the ratio of a horizontal axis to a vertical axis.

Subsequently, the local surface heat flux may be given by

$$q^*=\begin{cases}(x/L_r)^{-1/2} & \text{for vertical plates} \quad (25a)\\ (x/3L_r)^{-1/2} & \text{for vertical cones pointing upward} \quad (25b)\\ \dfrac{\sin\phi}{[(1-\cos\phi)(\sin^2\phi+\gamma^2\cos^2\phi)]^{1/2}} & \text{for horizontal ellipses} \quad (25c)\\ \dfrac{\sin^2\phi}{[(\frac{1}{3}\cos^3\phi-\cos\phi+\frac{2}{3})(\sin^2\phi+\gamma^2\cos^2\phi)]^{1/2}} & \text{for ellipsoids} \quad (25d)\end{cases}$$

where

$$q^*\equiv\left(\frac{q_w L_r}{(T_s-T_w)k}\right)\Big/\left(\frac{K(\rho-\rho_G)gL_r}{\rho\alpha\nu}\right)^{1/2}[-\theta'(0)] \tag{25e}$$

For an illustrative purpose, the local surface heat flux distributions on ellipses and ellipsoids are presented in Figs 2(a) and 2(b) for three different values of γ, namely: $\gamma=0.5$, 1 and 2. Naturally, the distribution for a small γ (ie a slender body) exhibits a pattern similar to that of a vertical flat plate (namely, $q_w \propto x^{-1/2}$). The heat flux for a larger γ (>1), on the other hand, increases away from the front stagnation point, and attains a maximum, as the flow accelerates due to a significant streamwise increase in the driving body force, namely g_x. But, as the boundary layer thickens further, it decreases downstream, resulting in a nonmonotonical variation of the surface heat flux.

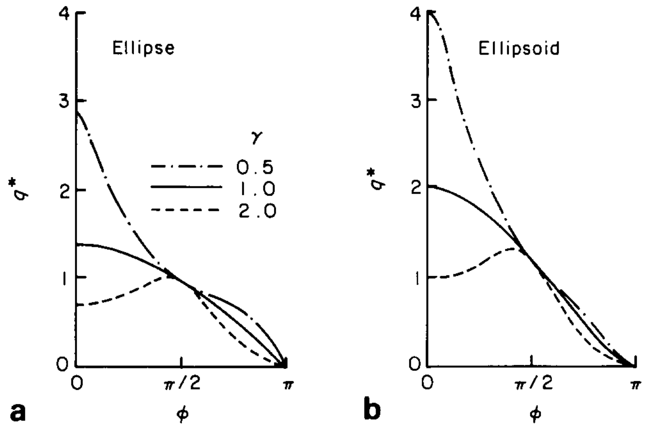

Figure 2 Geometric effect on local heat fluxes: (a) horizontal ellipses; (b) ellipsoids

It is also interesting to note that the q^* values at $\phi = \pi/2$ are at the same level for all values of γ. The observed heat flux distributions have very much in common with those in the single-phase free convection[4].

Concluding remarks

A general similarity transformation has been suggested for the analysis of film condensation and boiling within a porous medium. A similarity variable, which also considers the geometric effect on the boundary layer length scale, was proposed to deal with a two-dimensional or axisymmetric body of arbitrary geometrical configuration. As a result of this generalized similarity transformation, the governing equations, the boundary conditions and the matching conditions for a body of arbitrary geometry transform into those for a vertical flat plate, which have been previously solved by Cheng. Thus, the numerical values furnished for a flat plate are directly available for any particular geometry of concern.

Finally, it should be re-emphasized that the present results also apply to the case of film boiling simply by interchanging the roles of the vapour and the liquid, although the analysis was made specifically for the case of film condensation.

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Calendar

1987		
32nd ASME International gas turbine conference and exhibit	31 May–4 June 1987 Anaheim, California, USA	IGTC, 4250 Perimeter Park South, ∊ 108 Atlanta, Georgia, 30341, USA
5th Conference on Thermogrammetry and thermal engineering	8–10 June 1987 Budapest, Hungary	MATE Secretariat, H 1372 Budapest POB 451, Hngary
Twentieth Midwestern mechanics conference (20th MMC)	31 August–2 September 1987 Indiana, USA	Professor Hamilton, School of Mechanical Engineering, Purdue University, West Lafayette, Indiana 47907, USA
1987 ASME cogen-turbo	2–4 September 1987 Montreux, Switzerland	IGTC, 4250 Perimeter Park South, ∊ 108 Atlanta, Georgia 3041, USA
6th turbulent shear flow symposium	7–9 September 1987 Toulouse, France	Professor F. W. Schmidt, Secretary, Dept of Mechanical Engineering, The Pennsylvania State Univ., University Park, PA 16802, USA
Second international conference on laser anemometry	21–23 September 1987 Georgia, Scotland, UK	Dr S. M. Fraser, LDA Conference, Mechanical and Offshore Engineering, University of Strathclyde, Glasgow G1 1XJ, UK
International symposium on natural circulation	15–20 November 1987 New York, NY, USA	Dr J. H. Kim, Electric Power Research Institute, PO Box, 10412, Palo Alto, California 94303, USA
1988		
8th International Symposium on Fluid Power	18–21 April 1988 Birmingham, UK	Conference Dept. BHRA, The Fluid Engineering Centre, Cranfield, Bedford MK43 0AJ, UK
2nd International Conference on Flow Measurement	9–11 May 1988 London, UK	Conference Organizer, 2nd Flow Measurement BHRA, The Fluid Engineering Centre, Cranfield, Bedford MK43 0AJ, UK
Introduction to Flow Measurement Course	9–11 May 1988 London, UK	Mrs Jonna Foote-Kortland, Short Course Manager, Dept of Fluid Engineering and Instrumentation, Cranfield Institute of Technology, Cranfield, Bedford MK43 0AJ, UK
European Community wind energy Conference and Exhibition	6–10 June 1988 Herning, Denmark	H. S. Stephens & Associates, Conference Organizers, Agriculture House, 55 Goldington Road, Bedford MK40 3LS, UK